**Part b)**

In this section the three liquidity factors and the market excess returns are regressed against the excess returns of the portfolios. The two steps Fama-Macbeth method was used to test this model.

In the first step we estimate the betas for each portfolio. Then we estimate the risk premiums (gammas) by regressing the estimates of the betas on the excess returns of the portfolios.

The values of the gammas for the extended factors model are reported below:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | gamma0 | gamma1 | gamma2 | gamma3 | gamma4 |
| Extended facotr model | 0.2181  (0.6054) | 0.3069  (0.9631) | 0.0720  (4.6196) | 0.0256  (2.4805) | -0.0549  (-6.0122) |

Gamma0 has a value of 0.2181 and a t-value of 0.6054. Furthermore, when comparing the Gamma0 of the CAPM and the extended factors, the extended factors model produce lower gamma0 value. However, with a 5% significance level Gamma0 is insignificant.

The estimates for gamma1 is different from CAPM. Gamma1 has a value of 0.3069 and a t-value of 0.9631. With a 5% significance level Gamma1 is insignificant. Over time, Gamma1 appears to be positive in some cases and negative in others. This suggest that investors are sometimes compensated for added risk and penalized in other. The fact that Gamma1 takes a negative value can be explained by the fact that when the market returns are positive, portfolios with betas higher than 1 will may have lower returns as they are riskier.

The risk premium of the aggregate liquidity factor and innovation in liquidity are both positive and statistically significant. Therefore, portfolios with higher exposure to the liquidity factor are expected to have higher returns. This means investors are compensated for add risk. In other words, liquidity risk is priced. However, the risk premium for the traded liquidity in negative which contracts the results obtain for the risk premiums for the first two liquidity factors. A negative liquidity risk premium suggests that the liquidity risk is not priced.

Next, we estimate each liquidity factor in a separate regression as to avoid multi-collinearity. Since, the liquidity factors are correlated this may introduce multi-collinearity which affect the estimate of the gammas. First, we estimate the extended factor model with the aggregate liquidity factor only with the following regression:

Where beta 2 is the exposure of the portfolios to the aggregate liquidity.

There result of the regression is reported in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | gamma0 | gamma1 | gamma2 |
| aggregate liquidity | 0.866  (1.363) | -0.397  (-0.628) | 0.061  (2.515) |

The risk premium for the aggregate liquidity estimated in this regression does not seem to change by much from the previous regression. The values of gamma0 and gamm1 obtained from regression (2) is different from regression (1), however, the values are not statistically significant. The results from regression (2) points to the same conclusion as the previous one, that the liquidity risk is priced.

* Providing economic justification of the difference between the performance of CAPM and the extended factor model.

In the CAPM model the liquidity factors are not included. Hence, it is assumed in the CAPM that the only determinant of excess returns is the exposure to systematic risk. However, from an economic point of view, other factors like liquidity may predict the changes in excess returns. For example, a portfolio with a high exposure to liquidity risk can be undesirable by investors. That is because investors prefer more liquid assets to those that are less liquid. Hence, the demand for portfolios with high liquidity risk may be low. Therefore, the market will offer the investor higher return as to compensate them for the added risk.

Therefore, the extended factor model makes more realistic assumption about the determinants of the excess returns. By omitting the liquidity factor from the CAPM model, the model may suffer from the omitted variable problem. Also, the fact that liquidity is correlated with the excess market returns, when the market is preforming badly the liquidity available will decline significantly, leads to the CAPM to preform worse than the extended factor model. (reference1).

**Part c)**

In this section the CAPM and the extended factor model are re-estimated using data from the period September 2008 to September 2018. The results of the re-estimates are represented in table(x).

|  |  |  |  |
| --- | --- | --- | --- |
|  | gamma0 | gamma1 | gamma2 |
| capm | 0.926  (4.33) | -0.004  (-0.023) |  |
| Factor 1 | 1.28  (6.16) | -0.345  (-1.94) | -0.014  (-2.81) |
| factor 2 | 1.13  (5.89) | -0.24  (-1.41) | -0.024  (-2.82) |
| factor 3 | 0.95  (4.73) | -0.013  (-0.07) | -0.002  (-0.005) |

For CAPM the intercept value is 0.926 and it is significant with a t-value of 4.33. While gamma1 has a value of -0.004 and it is not significant. However, in the in-sample model of CAMP the estimate of the intercept is 1.35 with a t-value of 2.33. While, the gamma1 is -0.76 and it is insignificant with a t-value of -1.32. The out-of-sample CAPM has a lower intercept than the in-sample. However, the difference is not of a significant nature.

For the extended factor model, we ran three regressions for each factor. In the out-of-sample model, the intercept of the models with the aggregate liquidity and traded liquidity have increase from 0.86 and 0.88 to 1.28 and 0.95 respectively. The model with innovation in liquidity has decreased from 1.46 to 1.13.

In the out-of-sample data all of the three liquidity factors seem to have a negative gamma value. The out-of-sample estimation shows that both the aggregate liquidity and innovation in aggregate liquidity do predict changes in excess returns. However, it does not provide evidence for where the traded liquidity predicts the changes in excess returns or not.

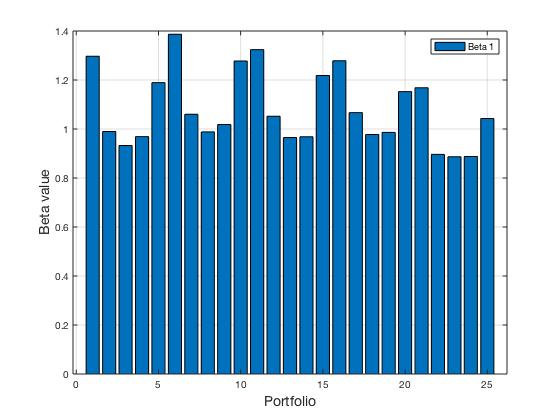
Similar to the in-sample estimation, the out-of-sample provides negative gamma value for the innovation in liquidity and the traded liquidity. Unlike in the out-of-sample, the in-sample three factor model provide positive gamma value for the aggregate liquidity factor.

The out-of-sample extended factor models implies that the liquidity risk is not price. Since all of the liquidity factors gammas are negative, this implies that a higher exposure to liquidity risk leads to lower returns. However, in the in-sample estimation of the risk premium of the aggregate liquidity was positive. This change may be explained by the downward spike in the gamma relating to aggregate liquidity in the out-of-sample data in 2009. The downward spike coincides with the credit crunch caused by the Financial crisis. In the Financial crisis, portfolios with higher sensitivity to liquidity suffered lower returns.

Generally speaking, the evidence shows that the liquidity factors do explain partially the movement in the excess returns of the portfolios in both the out-of-sample and in-sample. However, the out-of-sample results suggest that the liquidity factors are not priced. Compared to the out-of-sample, where the in-sample estimates provide mixed results.

\*\*

(The theory of asset pricing tells us that the gamma1 must be positive in expectation only. The fact that gamma1 take both positive and negative vales is driven by the volatility of the portfolios whose betas are greater than 1. Portfolios whose betas are greater than 1 exhibit higher fluctuations in their returns. For example, in a month were the riskier portfolios have lower returns than less risky portfolios, the gamma1 obtain in this month will be negative. This can be seen from the graph of gamma1’s over time. These fluctuations make it difficult to pin down the relationship between the betas and excess return of the portfolios. Also, the values of beta1, the exposure of the portfolios to factor1, is shown on figure1. Beta1 of the 25 portfolios range from 0.89 to 1.36. This may be problematic when calculating the values of the risk premium for the market excess returns. A low variation in the sample may cause estimation errors. However, the Betas for factor 2,3 and 4 have more variability.) (Low momentum = high beta).



Hameed, A., Kang, W. and Viswanathan, S., 2010. Stock market declines and liquidity. *The Journal of Finance*, *65*(1), pp.257-293.